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ANALYSIS OF MELTING BOUNDARY LAYERS ON DECELERATING BODIES

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SUMMARY

The flow of the viscous layer of ablating material on the surface of a body of revolution entering the atmosphere was investigated primarily in regard to phenomena over the entire body, in order to find the primary effect of the decelerating force on the flow and heat transfer. The phenomena were shown to be essentially unsteady exclusive of the region near the forward stagnation point. The significant dynamical parameters were determined and some solutions were obtained for various deceleration rates and times. These solutions show that deceleration causes an accumulation of ablating material in a region downstream of the stagnation point, and for this reason the thin-boundary-layer approximation will eventually fail to be appropriate.

INTRODUCTION

The ablation of skin material has been extensively studied as a means of reducing the heat transfer to reentry-type vehicles. For the most part analytical studies have been restricted to consideration of the stagnation region of bodies. On the basis of such an analysis (ref. 1), it was shown that for materials which melt before they vaporize, the presence of the liquid layer cannot be ignored in evaluating the characteristics of this type of thermal shield.

Although the liquid layer can be subject to a strong body force because of accelerations and decelerations of the vehicle, its effects on the flow and heat transfer have not been investigated in general. In determining conditions away from the stagnation region, it is particularly important to include the body force effects, because for vehicle decelerations the body-force opposes the downstream flow of liquid; in fact, it was recently pointed out in reference 2 that under certain conditions the liquid will be forced upstream and can eventually accumulate at some position away from the nose of the body. A qualitative discussion of deceleration effects is given in reference 3 and a similarity solution of the special case of a liquid layer subject to deceleration on a surface in a quasi-steady incompressible constant-pressure-gradient stream is presented in reference 4. Integral methods have been used to study the problem about more general bodies in references 5 and 6. In reference 2 it was pointed out that some important aspects of the problem were omitted in the analyses of references 4 and 5. In particular, the accumulation of liquid was precluded by the similarity assumption in reference 4 and by the assumed velocity profile in reference 5. Although the basic equations in reference 6 contain a deceleration term, no discussion of its significance is given therein; it will be shown subsequently that in a steady-state analysis of the problem, such as reference 6, no solutions containing significant deceleration effects can be obtained.

The purpose of this report is to extend the two-dimensional considerations of reference 2 to axisymmetric bodies and to present the principal novel features of the flow and heat transfer that result from a general treatment of the deceleration effects.

ANALYSIS

Conditions of Problem

The specific problem to be analyzed is the flow and heat transfer of a glassy viscous film of ablating material on the exterior surface of a body of revolution or symmetric two-dimensional body that enters the atmosphere at high speed and that experiences a large deceleration and surface heating. The viscosity of the liquid layer increases from some value at the gas-liquid interface to very large values near the body because of the temperature change. Density, specific heat, and thermal conductivity are assumed constant. For suitable materials and expected physical conditions the thickness of the region where the viscosity is low enough for the ablating material to be considered as fluid is very small compared with the body scale.

Some additional assumptions are made in the case to be analyzed in detail in order to show the physical phenomena most clearly and simply. In particular, the body is assumed to be subjected to a constant deceleration, although in an actual case the trajectory will determine the deceleration rate. Furthermore, the temperature at the gas-liquid interface is assumed constant, and the vaporization rate will be neglected; it will later be indicated how these restrictions might be relaxed for a more realistic calculation.

Because the liquid-layer thickness is small compared with the radius of curvature of the body, a system of coordinates parallel to and normal to the gas-liquid interface can be considered as a Cartesian coordinate system (see fig. 1). The interface is taken to be the surface $y = 0$, and y increases into the liquid. The acceleration terms resulting from the unsteady motion of the interface relative to the body are neglected, but the velocity is considered steady at any instant.

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The resulting equations of motion for the liquid layer are:

Continuity:

$$\frac{\partial}{\partial X} R^{\epsilon} U + \frac{\partial}{\partial Y} R^{\epsilon} V = 0 \tag{1}$$

where $\epsilon = 0$ for two-dimensional bodies and $\epsilon = 1$ for axisymmetric bodies.

Momentum:

$$\rho\left[\frac{\partial U}{\partial \bar{t}} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} - \sqrt{1 - \left(\frac{dR}{dX}\right)^2}\, A\right] = -\frac{\partial P}{\partial X} + \frac{\partial}{\partial Y}\left[\bar{\mu}\left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)\right] + 2\frac{\partial}{\partial X}\left(\bar{\mu}\frac{\partial U}{\partial X}\right) \tag{2}$$

$$\rho\left(\frac{\partial V}{\partial \bar{t}} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} - \frac{dR}{dX} A\right) = -\frac{\partial P}{\partial Y} + 2\frac{\partial}{\partial Y}\left(\bar{\mu}\frac{\partial V}{\partial Y}\right) + \frac{\partial}{\partial X}\left[\bar{\mu}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)\right] \tag{3}$$

Energy, with thermal expansion of the liquid neglected:

$$\rho c_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y}\right) = k\left(\frac{\partial^2 \bar{T}}{\partial X^2} + \frac{\partial^2 \bar{T}}{\partial Y^2}\right) + \Phi \tag{4}$$

Dissipation:

$$\Phi \equiv \bar{\mu}\left[2\left(\frac{\partial U}{\partial X}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2 + 2\frac{\partial U}{\partial Y}\frac{\partial V}{\partial X} + \left(\frac{\partial U}{\partial Y}\right)^2 + \left(\frac{\partial V}{\partial X}\right)^2\right]$$

(All symbols are defined in appendix A.) The transformation from stationary coordinates to accelerating coordinates fixed in the body is equivalent to the introduction of an equivalent body force $\rho\vec{A}$ per unit volume with the components of equations (2) and (3).

Scaling and Reduction of Equation

In order to compare the various terms and to determine their relative magnitudes, all variables will be transformed to dimensionless variables in such a way that they are of order 1; the magnitudes of the terms will then be indicated by the fixed coefficients.

For X and R the clear choice of scale is the body scale size L, so that

$$X = xL; \qquad R = rL$$

Scaling the pressure, temperature, and viscosity by the values at the stagnation point $(X = 0)$ interface $(Y = 0)$ gives

$$\overline{T} = \overline{T}_0 T; \qquad P = P_0 p; \qquad \overline{\mu} = \overline{\mu}_0 \mu$$

The scales for Y, $\overline{t}$, U, and V will be unspecified for the present but indicated by $L\delta$, σ, W, and F, respectively, so that

$$Y = L\delta y; \qquad \overline{t} = \sigma t; \qquad U = Wu; \qquad V = Fv$$

The scale $L\delta$ is related to the liquid-layer thickness. From the continuity equation,

$$F = \delta W$$

The first momentum equation (2) is employed to estimate the magnitude of W: Because the viscous fluid attains only small velocities, it is anticipated that the inertia terms can be neglected and that the shear and pressure forces are of the same order of magnitude. Thus

$$\frac{P_0}{L} = \frac{\overline{\mu}_0 W}{L^2 \delta^2}$$

or

$$W = \left(\frac{P_0 L}{\overline{\mu}_0}\right)\delta^2; \qquad F = \left(\frac{P_0 L}{\overline{\mu}_0}\right)\delta^3$$

Because the inertia terms are small, the time scale L/W obtained from the first term of the inertia equations is inadequate. The only remaining term for selection of the time scale is that of the energy equation. This is the most important unsteady effect, since the heating of the liquid will determine the rate of softening and hence the rate of velocity

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increase. Because of the slow motion, the rate of temperature increase should be balanced by the conduction term. Therefore

$$\sigma = \left(\frac{\rho c_p L^2}{k}\right)\delta^2$$

The remaining scale factor δ will be subsequently identified. In terms of the new variable the equations are

$$\frac{\partial}{\partial x} r^{\epsilon}u + \frac{\partial}{\partial y} r^{\epsilon}v = 0 \tag{5}$$

$$\frac{1}{Pr}\left[\frac{\partial u}{\partial t} + RePr\delta^2\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\right] - g\sqrt{1 - \left(\frac{dr}{dx}\right)^2} = -\frac{\partial p}{\partial x} + 2\delta^2\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}\right)\right] \tag{6}$$

$$\frac{\delta^2}{Pr}\left[\frac{\partial v}{\partial t} + RePr\delta^2\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\right] - \delta\frac{dr}{dx}g = -\frac{\partial p}{\partial y} + 2\delta^2\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + \delta^2\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}\right)\right] \tag{7}$$

$$\frac{\partial T}{\partial t} + RePr\delta^2\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \delta^2\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial^2 T}{\partial y^2} + \frac{PrW^2}{c_pT_0}\left\{\left(\frac{\partial u}{\partial y}\right)^2 + 2\delta^2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2\right] + \delta^4\left(\frac{\partial v}{\partial x}\right)^2\right\} \tag{8}$$

The small terms are deleted from these equations on the assumptions that $\delta \ll 1$, $Re\delta^2 \ll 1$, $Pr \gg 1$, and $Pr\,\frac{W^2}{c_pT_0} \ll 1$. The analysis herein will be applied to sudden heating of the interface, where $\partial T/\partial t$ is initially indefinitely large at $y = 0$. Because of the small extent of the region of softened material, however, $\partial u/\partial t$ and $\partial v/\partial t$ are not indefinitely large. The assumptions thus yield

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} - g\sqrt{1 - \left(\frac{dr}{dx}\right)^2} \equiv f(x) \tag{9}$$

$$\frac{\partial p}{\partial y} = \delta g \frac{dr}{dx} \sim 0 \tag{10}$$

$$\frac{\partial T}{\partial t} + \beta\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} \tag{11}$$

as well as the unaltered continuity equation (5).

The fact that the main unsteady effect in the ablation process is due to the unsteady term in the energy equation was also mentioned after a qualitative discussion in the stagnation-point analysis of reference 7. Because of equation (10), p is assumed constant through the liquid layer at any fixed station X on the body and is equal to its value at the interface ($p = p(X) = p_i(X) = p(X,0)$). The Newtonian pressure distribution is used for $p(X)$. The importance of deceleration is seen to depend on the magnitude of the parameter g (i.e., $\equiv A\rho L/P_0$), which represents the ratio of the deceleration body force to the pressure force. From Newtonian fluid mechanics the decelerating force on the body is of order $P_0 S$, where S is the projected cross-sectional area of the body. If LS is the body volume and $\rho_B LS$ the mass (ρ_B is average mass density), then the deceleration is

$$A = \frac{P_0}{L\rho_B} \qquad \text{or} \qquad g = \frac{\rho}{\rho_B} \tag{12}$$

Thus for a reentering rocket, g can be very large, whereas for a meteorite, g is of order 1. The second parameter β that appears in equation (11) indicates the importance of heat convection relative to heat conduction and depends on shear stress as well as on properties of the liquid layer.

The initial conditions of the body for the sudden application of boundary-layer heating are determined by the assumption of a cold glassy layer. The initial temperature is assumed to be 0, so that

$$t = 0; \qquad T = u = v = 0 \tag{13}$$

At the interior of the body ($y \to \infty$), the temperature remains low, but the melting away of the liquid layer results in a relative velocity v_∞ between the interface and the body. Therefore

$$y \to \infty; \qquad T = u = 0; \qquad v = v_\infty \tag{14}$$

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If $v_\infty < 0$, the glassy liquid is being carried away; if $v_\infty > 0$, the liquid is accumulating. Because of symmetry at the stagnation point,

$$x = 0; \qquad \frac{\partial T}{\partial x} = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0 \tag{15}$$

At the interface, the temperature and shear stress of the liquid are equal to those in the gas:

$$y = 0; \qquad \overline{T}_g = \overline{T}_i; \qquad \overline{\tau}_g = \overline{\tau}_i = -\overline{\mu}_i\left(\frac{\partial U}{\partial Y}\right)_i = \frac{\overline{\mu}_i W}{L\delta\mu_i}\,\tau_i = P_0\delta\tau_i \tag{16}$$

And finally the heat-balance condition is

$$k_g\left(\frac{\partial \overline{T}_g}{\partial Y}\right)_i = k\left(\frac{\partial \overline{T}}{\partial Y}\right)_i + \rho V_i H \tag{17}$$

as a restatement of equation (18) of reference 8. The convection of enthalpy by diffusion has been neglected, since a noncatalytic wall is assumed. Also, the radiation terms in the energy equation that are included in reference 6 have been neglected.

If μ is a known function of x and y, equation (9) can be integrated for u to yield

$$\mu\,\frac{\partial u}{\partial y} = -\tau_i + fy \tag{18}$$

$$u = f\int_\infty^y \frac{y}{\mu}\,dy - \tau_i\int_\infty^y \frac{dy}{\mu} \tag{19}$$

If some dependence of μ on y is assumed, equation (19) will yield a solution for u, which when inserted into the continuity equation (5) results in a first-order differential equation in x of the boundary conditions with explicit dependence of y. The variation with x thus originates in conditions in the gaseous boundary layer. The integral of equation (5) is

$$v - v_\infty = -\frac{1}{r^\epsilon}\frac{\partial}{\partial x}\left(r^\epsilon\int_\infty^y u\,dy\right) \tag{20}$$

$$v_i - v_\infty = -\frac{1}{r^\epsilon}\frac{\partial}{\partial x}\left(r^\epsilon \int_\infty^0 u\, dy\right) \tag{21}$$

However, the viscosity depends on the temperature, which is found as a solution of a partial differential equation with x, y, and t as independent variables. Because of the complicated form for the convection terms, the general solution of the energy equation is difficult. In reference 6 an integral approach has been used that relates the boundary values and gives no profiles; in the present case, however, because of the requirement that some details of the structure of the liquid layer be found, it was considered desirable to simplify the differential equation and to solve this approximate form in detail. In references 1 and 9, in which the problem is analyzed at the stagnation point, where $\partial T/\partial x = u = 0$, and for steady-state conditions, the equation is simplified by setting $v = v_\infty$. In the present case, the same substitution is made with justification as follows: In the first approximation, the interface temperature $T_i(X,t)$ is assumed to vary only slowly with X. Only in the thin region where $T \sim T_i$ are there appreciable flows, so that the effect of convection in this region of nearly uniform temperature is small. The only important effect of convection is the transport of the high-temperature interface toward the body as the viscous liquid layer is swept away or evaporated. The energy equation with these assumptions reduces to

$$\frac{\partial T}{\partial t} + \beta v_\infty \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \tag{22}$$

A discussion of the errors arising from this approximation is given in appendix B.

The inadequacy of the steady-state approach for application over the whole body (suggested in ref. 9 and used in ref. 6) can be seen by integration of equation (22). For the steady state, there is obtained from equation (22)

$$\frac{dT}{dy} = \left(\frac{dT}{dy}\right)_i e^{\beta v_\infty y}$$

This equation shows that dT/dy is unbounded as $y \to \infty$ in regions of X where $v_\infty > 0$. Such regions can exist on decelerating bodies, as previously described in reference 2. At the stagnation region, $v_\infty < 0$, so that no difficulty arises either at $X \to 0$ or $A = 0$. Although the energy equation (22) from which this result is derived is of questionable accuracy near the interface, it closely describes conditions for large

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values of y. The physical interpretation of this result is as follows: If a long body under steady deceleration is imagined, it is clear that in the cylindrical part no pressure gradient exists and that because of the thick boundary layer the surface shear forces are small. Consequently, the dominating force is a deceleration force acting as a gravitational field on a layer of liquid clinging to the wall. This liquid will therefore slump forward. Because of the equivalence of conditions at all locations, the motion will approximate that of uniform layers of fluid sliding over each other. The continued application of heat will cause a growth in thickness of the thermal layer on this section of the body, so that steady-state conditions are never attained. Also, the forward slumping flow from the back region and the backward-swept flow from the front region will meet at some intermediate station where the fluid will continue to accumulate. (This result will be modified when the accumulation of material is sufficient to alter the pressure distribution.) This region will also not approach a steady-state condition. Near the forward stagnation point, a steady-state solution is nearly attained.

No details except results of the computations made in reference 6 are contained therein, but because of the reasons cited previously those numerical results could not have included deceleration effects. The fact that no discussion of these effects is given therein seems to substantiate this supposition.

Method of Solution

The boundary conditions for calculation of the liquid layer are not all known a priori. At the interface there must be a match of temperature, shear stress, heat flow, and mass evaporation rate. If a temperature distribution $T_i(x,t)$ is assumed, all other quantities may be calculated from solutions of gaseous boundary layer. Various methods of making this match can be used; in addition to the description of some of these methods contained in the previously mentioned references, a comprehensive discussion of this problem is presented in reference 8. In the present analysis the gas boundary-layer characteristics of reference 10 were used for the assumed Newtonian pressure distribution, and hence the calculations (but not the analysis) are restricted to the class of bodies for which those similarity solutions apply. A representative two-dimensional body of this class is shown in reference 2; for axisymmetric bodies the Mangler transformation is applied to permit use of the results of reference 10. Since this procedure of using exact similar gas solutions is rather lengthy and involved and is not so good as direct use of the tables of reference 10, which can be used for any body shape, the details will be omitted; the axisymmetric body of this class studied herein is shown in figure 2.

When the liquid-layer flow and heat-transfer equations are solved, a discrepancy may be expected to exist in the heat balance for each assumed interface temperature distribution. From several assumptions of the interface temperature distribution $T_i(X,t)$ it should be possible to find a distribution by interpolation for which the heat balance conditions are satisfied. This procedure could be applied at each instant of time by starting with the value at $X = 0$ and working downstream by integration of the continuity equation. For the problem considered herein, of sudden application of the hot gas, a selection of $\overline{T}(X,0,\overline{t}) = 4000^\circ$ F (and $T_i = 1.0$) was chosen in order to permit a solution that would indicate the main kinematic features of the liquid glass layer as a whole and to show time and x-variation of the heat-flux parameter $(\partial T/\partial y)_i$. It was also assumed that there is no evaporation, that is,

$$v_i = v(x,0,t) = 0$$

The energy equation is integrated directly to give

$$T = \frac{T_i}{2}\left[e^{\beta v_\infty y} \operatorname{erfc} \frac{1}{2}\left(\frac{y}{\sqrt{t}} + \beta v_\infty \sqrt{t}\right) + \operatorname{erfc} \frac{1}{2}\left(\frac{y}{\sqrt{t}} - \beta v_\infty \sqrt{t}\right)\right] \qquad (23)$$

for the assumption that both T_i and v_∞ are independent of time; these assumptions are more realistic for t large than for initial conditions.

Completing the solution requires an explicit form for the dependence of μ on y. For this purpose, assume

$$\mu = \mu_i \exp(ay + by^2)$$

The functions a and b are determined from the assumed dependence of viscosity on the temperature:

$$\mu = \mu_i\left(\frac{T}{T_i}\right)^{-n}$$

Differentiating yields

$$\frac{1}{\mu_i}\left(\frac{d\mu}{dy}\right)_i = a = -\frac{n}{T_i}\left(\frac{dT}{dy}\right)_i \equiv -nT'$$

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From equation (23) the required gradient T' is found for identifying a:

$$\frac{1}{T_i}\left(\frac{\partial T}{\partial y}\right)_i \equiv T' = \frac{1}{2}\left[\beta v_\infty \operatorname{erfc}\left(\frac{\beta v_\infty \sqrt{t}}{2}\right) - \frac{2}{\sqrt{\pi t}} e^{-\beta^2 v_\infty^2 t/4}\right] \tag{24}$$

The quadratic term is determined by the temperature at large values of y, where equation (23) is approximated by

$$\frac{T}{T_i} \sim \frac{2}{\sqrt{\pi}} \frac{\exp\left(\frac{-y^2}{4t} + \frac{\beta v_\infty y}{2} - \frac{\beta^2 v_\infty^2 t}{4}\right)}{\frac{y}{\sqrt{t}} - \frac{\beta^2 v_\infty^2 t^{3/2}}{y}}$$

The dominating factor in determining the rate of decrease of the temperature at large y is

$$\exp \frac{-y^2}{4t}$$

Hence $b = n/4t$ is chosen, from which

$$\mu = \mu_i \exp\left[-n\left(T'y - \frac{y^2}{4t}\right)\right] \tag{25}$$

With this viscosity relation, equation (19) may be explicitly integrated to

$$u = -\frac{\exp n\left(yT' - \frac{y^2}{4t}\right)}{n\left(T' - \frac{y}{2t}\right)}\left\{\frac{f}{\mu_i}\left[\frac{\theta}{nT'\left(1 - \frac{y}{2tT'}\right)^2} - y\right] + \frac{\tau_i}{\mu_i}\left[1 - \frac{\theta}{2n\left(\frac{y}{2\sqrt{t}} - T'\sqrt{t}\right)^2}\right]\right\} \tag{26}$$

where the abbreviations are

$$\theta \equiv 2Z^2\left(1 - \sqrt{\pi}\, Ze^{Z^2} \operatorname{erfc} Z\right)$$

$$Z \equiv \sqrt{n}\left(\frac{y}{2\sqrt{t}} - T'\sqrt{t}\right)$$

Also, by integration of the continuity equation,

$$v - v_\infty = \frac{e^{-ny^2/4t}}{n^2 r^\epsilon} \frac{\partial}{\partial x}\left(\frac{r^\epsilon e^{nyT'}}{\left(T' - \frac{y}{2t}\right)^2}\left\{-\frac{f}{\mu_i}\left[y + \frac{\theta}{n\left(T' - \frac{y}{2t}\right)} + 2tT'(\theta - 1)\right] + \frac{\tau_i \theta}{\mu_i}\right\}\right) \tag{27}$$

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At the interface $v = v_i$, $Z = -\sqrt{nt}\, T' \equiv Z_i$, $\theta = \theta_i$, and

$$v_i - v_\infty = \frac{1}{n^2 r^\epsilon}\frac{\partial}{\partial x}\left(\frac{r^\epsilon \theta_i}{\mu_i T'^2}\left\{-f\left[\frac{1}{nT'} + 2tT'\left(\frac{\theta_i - 1}{\theta_i}\right)\right] + \tau_i\right\}\right) \tag{28}$$

At the stagnation point $r = x$, $f = x(df/dx)_0$, $\tau_i = x(d\tau_i/dx)_0$, $\mu_i = 1$, $T' = T_0'$, and θ_i = constant, so that

$$u_i(0) = -\frac{x}{nT'}\left[\frac{\theta_i}{nT_0'}\left(\frac{df}{dx}\right)_0 + \left(\frac{d\tau_i}{dx}\right)_0\left(1 + \frac{\theta_0}{2ntT'^2}\right)\right] \tag{26a}$$

$$v_i(0) - v_\infty(0) = \frac{1 + \epsilon}{n^2 T_0'^2}\,\theta_0\left[-\left(\frac{df}{dx}\right)_0\left(\frac{1}{nT_0'} + 2tT_0'\,\frac{\theta_0 - 1}{\theta_0}\right) + \left(\frac{d\tau_i}{dx}\right)_0\right] \tag{28a}$$

The limiting steady-state case is obtained when t is large, $Z_i = -\sqrt{nt}\, T'$ is large, and

$$\theta \sim 1 - \frac{1.5}{Z^2} \rightarrow 1 - \frac{1.5}{nt\left(T' - \frac{y}{2t}\right)^2} \rightarrow 1 - \frac{1.5}{ntT'^2}$$

At this point the problem is completely solved for dependence of u, v, and T on x, y, and t, provided that the dependence of the boundary parameters v_∞, T', T_i, v_i, and τ_i on the variables X and t is found. For the approximate solution, $v_i = 0$, but, in general, the temperature balance $\overline{T}_i = \overline{T}_g$ will determine the vapor pressure of the components of the liquid glass, and the diffusion rate through the boundary layer (see refs. 9 and 11) will depend on the external conditions and the wall temperature. Similarly, the shear stress and heat

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transfer will depend on T_i and external conditions. The terms T' and v_∞ are left to be found from equations (24) and (28).

Scale of δ

Since the equations are available for a solution, it is possible at this point to find an approximate, particular solution from which a reasonable selection can be made for the arbitrary thickness parameter δ, the ratio of y scale to x scale. This estimate is made in appendix C.

Numerical Procedure

In order to obtain the parameters v_∞ and T', which are required before u, v, and T can be calculated, equations (24) and (28) must be solved simultaneously. The discussion is facilitated by writing the differential equation (28) in the form

$$v_\infty - v_i = \frac{1}{r^\epsilon}\frac{\partial}{\partial x} r^\epsilon(Bf + C\tau_i) \tag{29}$$

where B and C are functions of T' and t. On differentiation,

$$v_\infty - v_i = B\left(\frac{df}{dx} + \epsilon\,\frac{f}{r}\,\frac{dr}{dx}\right) + C\left(\frac{d\tau_i}{dx} + \frac{\epsilon\tau_i}{r}\,\frac{dr}{dx}\right) + \left[\left(f\,\frac{\partial B}{\partial T'} + \tau_i\,\frac{\partial C}{\partial T'}\right)\frac{\partial T'}{\partial v_\infty}\right]\frac{dv_\infty}{dx} \tag{30}$$

In the particular problem solved (the first approximation where $\overline{T}_i = \overline{T}_0$, $d\overline{T}_0/dt = 0$) the coefficient of dv_∞/dx was a small quantity; at $x = 0$, the conditions $f = \tau_i = 0$ cause the coefficient to vanish there. The usual integration procedure was therefore unsuitable in that successive approximations to the solution at a point frequently failed to converge. Equation (30) was therefore solved for the term v_∞ by writing the equation with numerical evaluation of the derivative from the argument itself:

$$v_\infty^i = D + E\,\frac{dv_\infty}{dx} = D + \frac{E}{\Delta x}\left(S^{i-2}v_\infty^{i-2} + S^{i-1}v_\infty^{i-1} + S^i v_\infty^i\right) \tag{31}$$

This method of solution was inadequate in an intermediate region for certain cases of large values of time and deceleration; the possible cause of this failure will be discussed in the RESULTS. In those cases it was possible to begin the solution at $x \to \infty$ and leave an intermediate region with the solution undetermined. For large values of x, it was assumed that $v_\infty = dv_\infty/dx = 0$.

RESULTS

In the example calculated, the ablating material was taken to be Pyrex and the conditions assumed were as follows:

Flight Mach number	18.0
Altitude, ft	90,000
Characteristic length, $L = R_0$, ft	1
Density, ρ, lb/cu ft	131
Conductivity, k, Btu/(ft)(°F)(sec)	1.71×10^{-3}
Specific heat (pressure constant), c_p, Btu/(lb)(°F)	0.29
Coefficient of viscosity at stagnation point, $\bar{\mu}_0$ (at 4000° F), slug/(ft)(sec)	0.07
Acceleration rate, A, gravity units (g's)	-70, -23.2

Body shape is shown in figure 2. From these conditions,

Prandtl number, Pr	383
Reynolds number, Re	79.6
Scaling factor, δ	2.510×10^{-3}
Heat-convection parameter, β	0.1929
Scaling factor, W, ft/sec	1.370
Scaling factor, F, ft/sec	3.446×10^{-3}
Dimensionless acceleration parameter, g	-0.6, -0.2

For Pyrex under the conditions of the problem, a value of n of 8 was assumed. The gaseous boundary layer adjoining the liquid layer was assumed to be laminar throughout its entire extent.

Development of the normal interface velocity v_∞ and the interface normal temperature gradient T' for the condition of no deceleration is shown in figure 3. The ablation velocity v_∞ indicates a steady increase in ablation rate at the stagnation point to a final value of $v_\infty \sim -1.25$ (corresponding to removal of material at the rate of 0.052 in./sec). Farther downstream, the material accumulates in a slight bump, which is swept downstream as a kind of single wave. This phenomenon may be understood to result from the decrease in shear stress with downstream distance; the backflow induced by the pressure increases with time because of the thickening of the softened layer on which the pressure acts. With increasing time, the temperature gradient decreases from a relatively high uniform value, as might be expected from the initial sudden application of $T = 1$ and relatively small convection, to a lower steady-state value near the stagnation region and zero value far back on the body. The temperature gradients for very short times cannot be accurate, because the boundary layer, as a result of thickening, will provide gradients that decrease as X increases, whereas the figure shows constant values. Conditions in the stagnation region are approximately steady state at $t = 29$ (corresponding to $\bar{t} = 4.1$ sec). At all times the most severe thermal load is imposed at the forward stagnation point. This occurrence is easily understood because (1) the thickness of the gaseous boundary layer is a mininum near the stagnation point, and (2) there is a large negative normal velocity, which results from the flowing away of material and which reduces the thickness of the thermal layer.

Details of the structure of the viscous layer are shown in figure 4. All temperature profiles are about the same for short time, but with increasing time the stagnation-point profile approaches a steady-state curve, whereas the others are all nearly the same as for unsteady heating of a slab. (After $X = 0$, the next profile is selected for such a value of X that $v_\infty = 0$; therefore, u_i is nearly a maximum there.)

Deceleration of the body causes changes in the behavior of the liquid layer as shown by comparison of figures 5 to 7 with previously mentioned results. The normal interface velocity at the stagnation point is reduced 6 percent for maximum deceleration. Farther back the calculations break down in a region where the normal interface velocity v_∞ exhibits large gradients. The inadequacy of the equations used herein to describe the condition in this region probably arises from the failure of the boundary-layer assumption because of the accumulation of fluid and the thickening of the liquid layer. The results of the present calculations show this region of large positive normal velocity v_∞, which results from the arrival of the fluid from the forward section by boundary-layer drag and from the slumping forward of material from the back end because of deceleration; at this location the forces balance. The accumulation of liquid into a bump may be directly inferred from the normal interface velocity v_∞ curves of figures 5(a) and 6(a). Definite values of v_∞ and the growth of the bump size cannot be given because of the failure of the backward and forward solutions to coalesce, but order of magnitude interpretation of the curves indicates the growth rate to be comparable with ablation rate at the stagnation region. Calculations could be made downstream of the critical region because of the small influence of the derivative and the resulting local character of the solution.

The failure of the solutions obtained by forward and rearward integration to match in the region of liquid accumulation is not surprising. In general, two asymptotic solutions (here for small and large distances from the stagnation point) cannot be joined without careful analysis. Sometimes the matching is further complicated because of the occurrence of a singularity in the intermediate region due to the omission of terms in the asymptotic equations that are significant there. Matching the asymptotic solutions properly in such a region requires that the analytic form of the solution there must be found. For the present problem there appears to be a distinct possibility of finding the solution of the Navier-Stokes equations in this accumulation region, because the inertia terms should be negligible there. Further consideration is being given to this point.

The interface temperature gradient approaches very small values at the bump because of the accumulation of the hottest liquid there. The temperature gradient at the stagnation point is reduced only 3.5 percent for maximum deceleration under the conditions of the calculations.

Both the velocity and the temperature profiles for high deceleration rates show clearly the dissimilarity of shape at various locations; there is even a case of flow reversal that results from the opposing effects of surface shear and body force. It is clear that the assumption of similar profiles as in reference 4 is unsuitable.

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A large accumulation of material in a bump will probably not be realized in a real situation because it would be ripped off by the airstream if it grows sufficiently large. The probability of this occurrence is enhanced for smaller bodies, as may be seen by finding the effect of the body size on the scaling factors. The normal velocity scale $F = (P_0 L/\overline{\mu}_0)\delta^3$ is independent of the body size, and therefore the relative magnitude of the bump varies inversely as the body size; thus on reduction of L by a factor of 100 to a diameter of 1/4 inch, the rate of growth of the bump of the order of 0.05 inch per second is very large. The body scale has an additional effect on the gaseous boundary layer in that the shear varies as $L^{-1/2}$. This variation tends to push the bump downstream and to increase the forward area from which material is accumulating as well as the flow rate u. These effects also enhance the rate of growth of the bump in smaller bodies as compared with larger ones. Since meteorites are dense, the deceleration rate is low ($g = O(1)$) according to equation (12) and will tend to reduce the effect.

The heat flow from the gas to the liquid was calculated at the stagnation point to be 53,200 Btu per square foot per second by the method of reference 12, and the results of reference 8 giving the ratio of heat-transfer rate over a hemisphere to that at the stagnation point were used to estimate the value elsewhere. If vaporization is neglected, the temperature gradient in the liquid at the interface is then 311,000° F per foot. On a dimensionless basis, the stagnation-point temperature gradient approaches the limit

$$T' = 311{,}000 \frac{\delta L}{T_i} = 0.1755$$

This value and those at several other locations are shown in figure 5. Thus, the heat load estimated from the liquid layer herein is too high. This error results from having taken too high a value for the interface temperature.

The temperature gradient of the liquid at the interface will depend on the ablation rate v_∞, which, through the viscosity, will depend strongly on the temperature. An assumption of a lower temperature will thus greatly reduce the interface temperature gradient and heat flow; closer agreement with other results (ref. 9) can then be expected.

For initial conditions (t small), the correction required is much greater; the corrected temperature will therefore rise from a low initial value to the final equilibrium value.

SUMMARY OF RESULTS

Analysis of the flow of a viscous layer of fluid on a body subjected to sudden atmospheric heating and deceleration yielded the following results:

1. Flow, temperature, and heat transfer in the liquid layer depended on the deceleration parameter, the heat-convection parameter, and the body shape, in addition to those quantities already found for the steady-state condition at the stagnation point.

2. A steady-state solution was possible only in the forward part of the body where the ablation process was removing rather than accumulating material. On the aft part an unsteady solution was required.

3. Similarity solutions were impossible; the velocity and temperature profiles varied radically in shape from one portion of the body to another and at different instants of time.

4. The heaviest heat load and ablation rate occurred at the stagnation point; deceleration affected these values slightly.

5. An accumulation of fluid occurred in the region where body, shear, and pressure forces were approximately balanced. This accumulation might cause a substantial change in the body shape for small bodies in which the fluid would be blown off a shoulder rather than flow off the back end.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, March 13, 1962

APPENDIX A

SYMBOLS

A bar over a symbol or use of a capital letter indicates a quantity with dimensions. Absence of the bar or use of a lower case letter indicates a dimensionless quantity - the dimensional quantity divided by an appropriate scale factor. Bars in this sense are omitted from this list.

A	acceleration rate of body, ft/sec^2; scaling factor, $P_0/\rho L$
B,C	notation quantity; compare eqs. (28) and (29)
c_p	specific heat of liquid (pressure constant), Btu/(slug)(°F)
D,E	notation quantity; compare eqs. (30) and (31)
F	scaling factor for V
f	dimensionless body force; $f = \frac{dp}{dx} - g\sqrt{1 - \left(\frac{dr}{dx}\right)^2}$
g	dimensionless acceleration parameter, $A\rho L/P_0$
H	heat of vaporization of liquid, Btu/slug
k	conductivity of liquid, (Btu)(ft)/(sq ft)(sec)(°F)
L	characteristic length of body, ft
n	index in viscosity-temperature relation $(\bar{\mu}/\bar{\mu}_0 = (\bar{T}/\bar{T}_0)^{-n})$
P	pressure, lb/sq ft; scaling factor P_0
Pr	Prandtl number of liquid, $c_p\bar{\mu}_0/k$
P_0	pressure at $X = 0$, lb/sq ft
p	dimensionless pressure, P/P_0
R	distance from axis of body to surface, ft; scaling factor, L
Re	Reynolds number of liquid layer, $\rho LW/\bar{\mu}_0$
r	dimensionless value of R; $r = R/L$

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S	projected cross-sectional area of body
S^i	weighting factor of value of function of X at $X = X_i$ for calculating derivative of function
T	dimensionless temperature, $\overline{T}/\overline{T}_0$
$\overline{T}$	temperature, $^{\circ}R$; scaling factor, $\overline{T}_0$
T'	$\left(\frac{1}{T}\frac{\partial T}{\partial y}\right)_i$
t	dimensionless time variable
$\overline{t}$	time, sec; scaling factor, $\sigma = (\rho c_p L^2/k)\delta^2$
U	velocity of fluid parallel to X, ft/sec; scaling factor, W
u	dimensionless U variable; $u = U/W$
V	velocity of fluid normal to body surface, relative to interface, ft/sec; scaling factor, $W\delta$
v	dimensionless V variable; $v = V/W\delta$
v_∞	ablation velocity at interface with respect to body, dimensionless
W	scaling factor for U; $W = (P_0 L/\overline{\mu}_0)\delta^2$, ft/sec
X	distance along body surface measured from stagnation point in plane containing body axis, ft; scaling factor, L
x	X/L
Y	distance normal to body surface measured from interface inward, ft; scaling factor, $L\delta$
y	$Y/L\delta$
Z	$\sqrt{n}\left(\frac{y}{2\sqrt{t}} - T'\sqrt{t}\right)$
β	heat-convection parameter, Pr Re δ^2
δ	scaling factor for distance normal to interface, $\delta = \left[(1 + \epsilon)\overline{\mu}_0^2(d\overline{\tau}/dX)_0 \big/ (\rho c_p/k)^2 n^2 L^3 P_0^3\right]^{1/9}$, dimensionless

ϵ 0 for two-dimensional problem, 1 for three-dimensional case

θ $2Z^2\left(1 - \sqrt{\pi}\, Ze^{Z^2} \operatorname{erfc} Z\right)$

μ coefficient of viscosity

ρ density of liquid, slug/cu ft

σ scaling factor for time, $\sigma = (\rho c_p L^2/k)\delta^2$, sec

$\overline{\tau}$ shear stress, lb/sq ft; scaling factor, $P_0\delta$

Φ dissipation function; scaling factor, $\overline{\mu}_0 W^2/L^2\delta^2$

Subscripts:

g gas

i interface between liquid and gas

0 stagnation point

APPENDIX B

MAGNITUDE OF ERRORS THAT ARISE IN THE APPROXIMATE ENERGY EQUATION

An error arises from the use of the approximate form of the energy equation (22) in place of the more accurate form (11). Physically this approximation corresponds to consideration of heat convection resulting from motion of the thermal layer as a whole, while the convection arising from details of the internal motion in the liquid layer is neglected. Because convection is unimportant in the initial period (t small) when the glassy layer is first beginning to soften, the discussion is limited to the steady-state condition with the assumption that $v_\infty < 0$. The method of this report (eq. (22)) is first applied to these special conditions, and then a parallel calculation is made for the more exact form, followed by an estimate of the effect of the differences.

The approximate form

$$T_{yy} = \beta v_\infty T_y$$

is integrated to

$$\log \frac{T_y}{(T_y)_i} = \beta v_\infty y$$

Then

$$T_i = \int_\infty^0 T_y \, dy = \frac{(T_y)_i}{\beta v_\infty}$$

from which the interface gradient is

$$T'_{\text{approx.}} \equiv \frac{(T_y)_i}{T_i} = \beta v_\infty$$

For the more exact form,

$$\log \frac{T_y}{(T_y)_i} = \beta \int_0^y \left(v + \frac{uT_x}{T_y} \right) dy$$

If it is assumed that T/T_i is a function of y/δ_T, where δ_T is the thickness of the thermal layer, and that the velocity layer thickness δ_v is proportional to δ_T, then with the further assumption that $\partial T_i/dx = 0$, equation (22) is integrated to

$$\log \frac{T_y}{(T_y)_i} = \beta \int_0^y \left(v - \frac{uy}{\delta_v} \frac{d\delta_v}{dx}\right) dy$$

The velocity layer thickness δ_v describes the region where u is of the order of u_i and it may be defined as

$$\delta_v^{-1} \equiv - \frac{1}{u_i}\left(\frac{\partial u}{\partial y}\right)_i$$

This quantity is distinguished from the constant scaling factor δ in that δ_v is a variable function of x and t, and is scaled by the length δL. Similarly δ_T may be defined in terms of the initial temperature gradient

$$\delta_T^{-1} \equiv -T'$$

For the estimate of the magnitude of the error the following approximations for the velocity layer $(y < \delta_v)$ are used:

$$u = u_i e^{-y/\delta_v}; \qquad v = v_\infty\left(1 - e^{-y/\delta_v}\right)$$

from which

$$\log \frac{T_y}{(T_y)_i} = \beta \left\{ v_\infty\left[y - \delta_v\left(1 - e^{-y/\delta_v}\right)\right] - \frac{u_i}{\delta_v} \frac{d\delta_v}{dx} \delta_v^2 \left[1 - \left(1 + \frac{y}{\delta_v}\right) e^{-y/\delta_v}\right]\right\}$$

Because the temperature gradient varies only slightly in the velocity layer,

$$\log \frac{T_y}{(T_y)_i} \sim \frac{T_y}{(T_y)_i} - 1$$

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which yields the approximation

$$T_y = (T_y)_i \left\{1 + \beta v_\infty\left[y - \delta_v\left(1 - e^{-y/\delta_v}\right)\right] - \beta u_i \frac{d\delta_v}{dx}\delta_v\left[1 - \left(1 + \frac{y}{\delta_v}\right)e^{-y/\delta_v}\right]\right\}$$

The temperature is obtained from the gradient by expanding in power series the exponentials in the preceding equation and retaining the quadratic terms. Integration of the resulting expression yields

$$T = T_i\left\{1 + T'\left[y + \left(v_\infty - u_i \frac{d\delta_v}{dx}\right)\frac{\beta y^3}{6\delta_v}\right]\right\}$$

For the region $y > \delta_v$, if the approximation $v = v_\infty$, $u = 0$ is used,

$$T_{yy} = \beta v_\infty T_y$$

$$T_y = Ke^{\beta v_\infty y}$$

$$T = \frac{k}{\beta v_\infty} e^{\beta v_\infty y} = \frac{T_y}{\beta v_\infty}$$

If values of T'/T from the inner and the outer solutions are now matched at the point $y = \delta_v$,

$$\beta v_\infty = \frac{T_y}{T} = \frac{T'\left[1 + \frac{\beta v_\infty \delta_v}{e} - \beta u_i \frac{d\delta_v}{dx}\delta_v\left(1 - \frac{2}{e}\right)\right]}{1 + T'\left[dv + \left(v_\infty - u_i \frac{d\delta_v}{dx}\right)\frac{\beta\ dv^2}{6}\right]}$$

$$\frac{T'_{approx.}}{T'} = 1 - \beta\delta_v\left[v_\infty\left(1 - \frac{1}{e}\right) + u_i\left(1 - \frac{2}{e}\right)\frac{d\delta_v}{dx}\right] - \frac{\beta^2\delta_v^2}{6} v_\infty\left(v_\infty - u_i \frac{d\delta_v}{dx}\right)$$

For the magnitudes involved, the term in β^2 may be neglected. The variation of the term in β is such that a maximum value can be expected at the stagnation point, since $u_i\ d\delta_v/dx$ is positive, going to zero at $x = 0$, and v_∞ is negative with a maximum magnitude there. Thus the maximum error (at $x = 0$) is

$$\frac{T'_{approx.} - T'}{T'} \sim -0.632\ \beta\delta_v v_\infty \sim \frac{0.632\ \delta_v}{\delta_T}$$

where the approximation

$$\delta_T = \frac{-1}{T'} \sim \frac{-1}{\beta v_\infty}$$

has been utilized.

Examination of the detailed calculations near $x = 0$ for no deceleration and for t large shows $\delta_V/\delta_T \sim 0.1$. Thus $T'_{approx.}$ is too large by about 6 percent. At $y = \delta_V$ the temperature has changed by approximately the ratio $\delta_V/\delta_T \sim 0.1$; consequently, the maximum error in T is 0.6 percent, and is less for $y \neq \delta_V$.

Equations (26a) and (28a) show that u_i has terms in $1/T'$ and $(1/T')^2$ which lead to errors of between 6 and 12 percent. Correspondingly, the ablation velocity is too small by an amount between 12 and 18 percent. All these errors are less at regions other than that near the front stagnation point and for shorter times when the velocities have not attained their maximum values. The errors are also reduced for materials having a larger index n of viscosity dependence on the temperature, since the hypothesis of uniform temperature in the convective velocity layer is more closely satisfied for such fluids.

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APPENDIX C

SCALE OF δ

The scale of δ is selected in such a way that the requirement $v_\infty = O(1)$ is satisfied. For this purpose $v_\infty = -1$ is chosen at $X = 0$ for $t = \infty$, on the assumption of v_i negligible and $(df/dX)_0 = 0$ (large radius of curvature at the stagnation point). With these assumptions, equation (24) reduces to

$$T_0' = -\beta = -\mathrm{PrRe}\delta^2$$

and equation (28a) to

$$T_0'^2 = \frac{1 + \epsilon}{\mu^2}\left(\frac{d\tau_i}{dX}\right)_0$$

From these two equations T_0' is eliminated. The involvement of δ in Re and $(d\tau_i/dX)_0$ is

$$\mathrm{Re} = \frac{\rho L}{\bar{\mu}_0} W = \frac{\rho L^2 P_0}{\bar{\mu}_0^2}\delta^2$$

$$\left(\frac{d\tau_i}{dx}\right)_0 = \frac{L}{P_0\delta}\left(\frac{d\bar{\tau}_i}{dX}\right)_0$$

With $\mu_{i,0} = 1$,

$$\delta^9 = \frac{1 + \epsilon}{\left(\frac{\rho c_p}{k}\right)^2}\frac{\bar{\mu}_0^2}{(LP_0)^3}\left(\frac{d\bar{\tau}_i}{dX}\right)_0$$

Then

$$\beta = \mathrm{Pr}\frac{\rho L^2}{\bar{\mu}_0^2}P_0\delta^4$$

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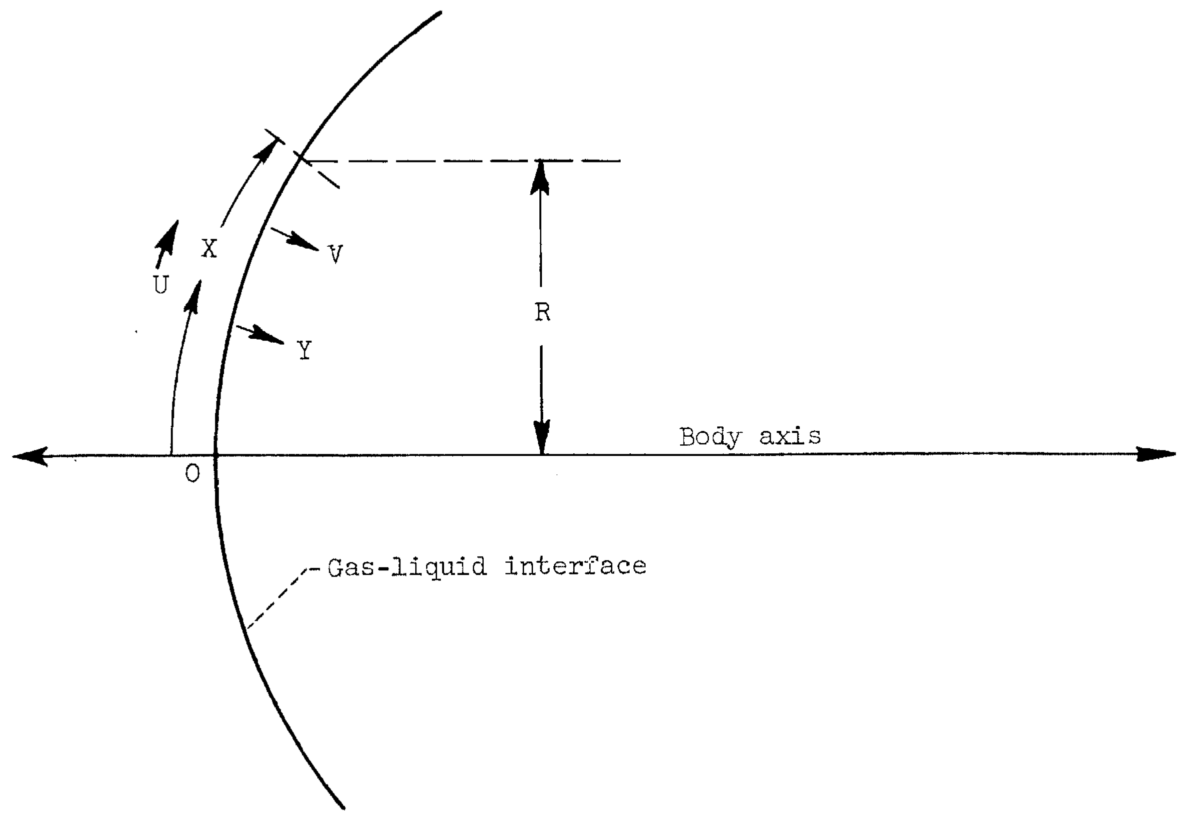


Figure 1. - Coordinate system.

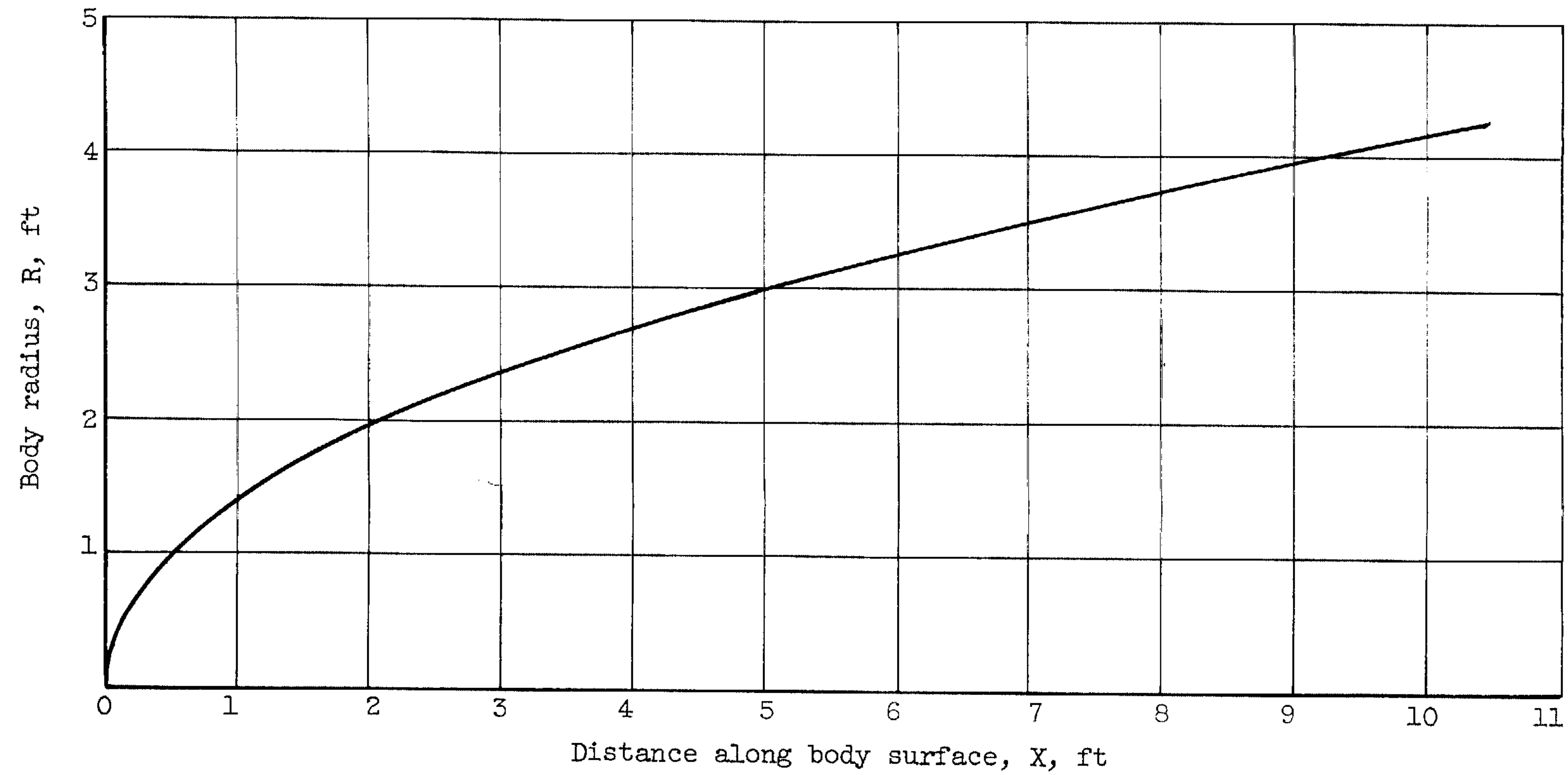
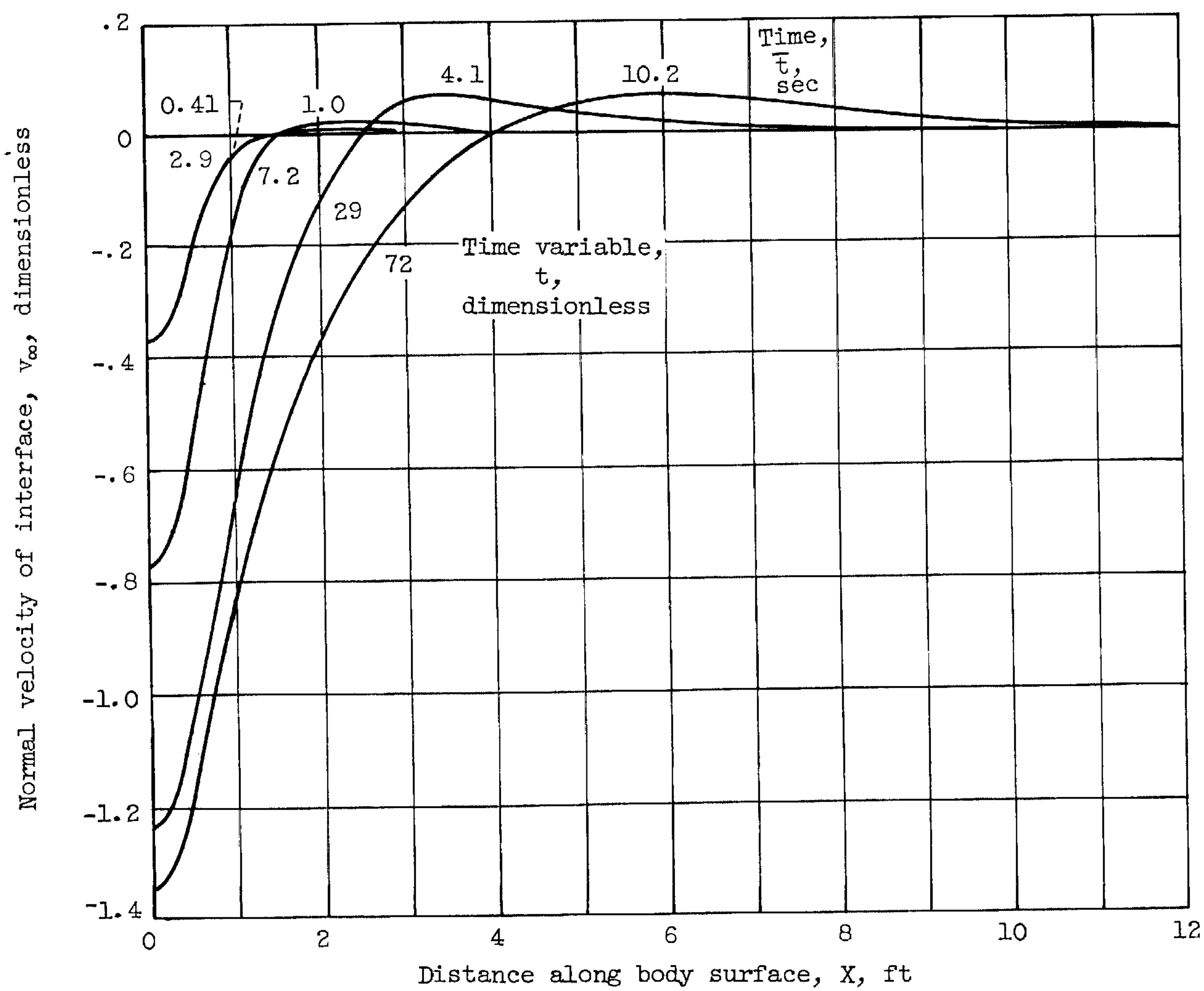
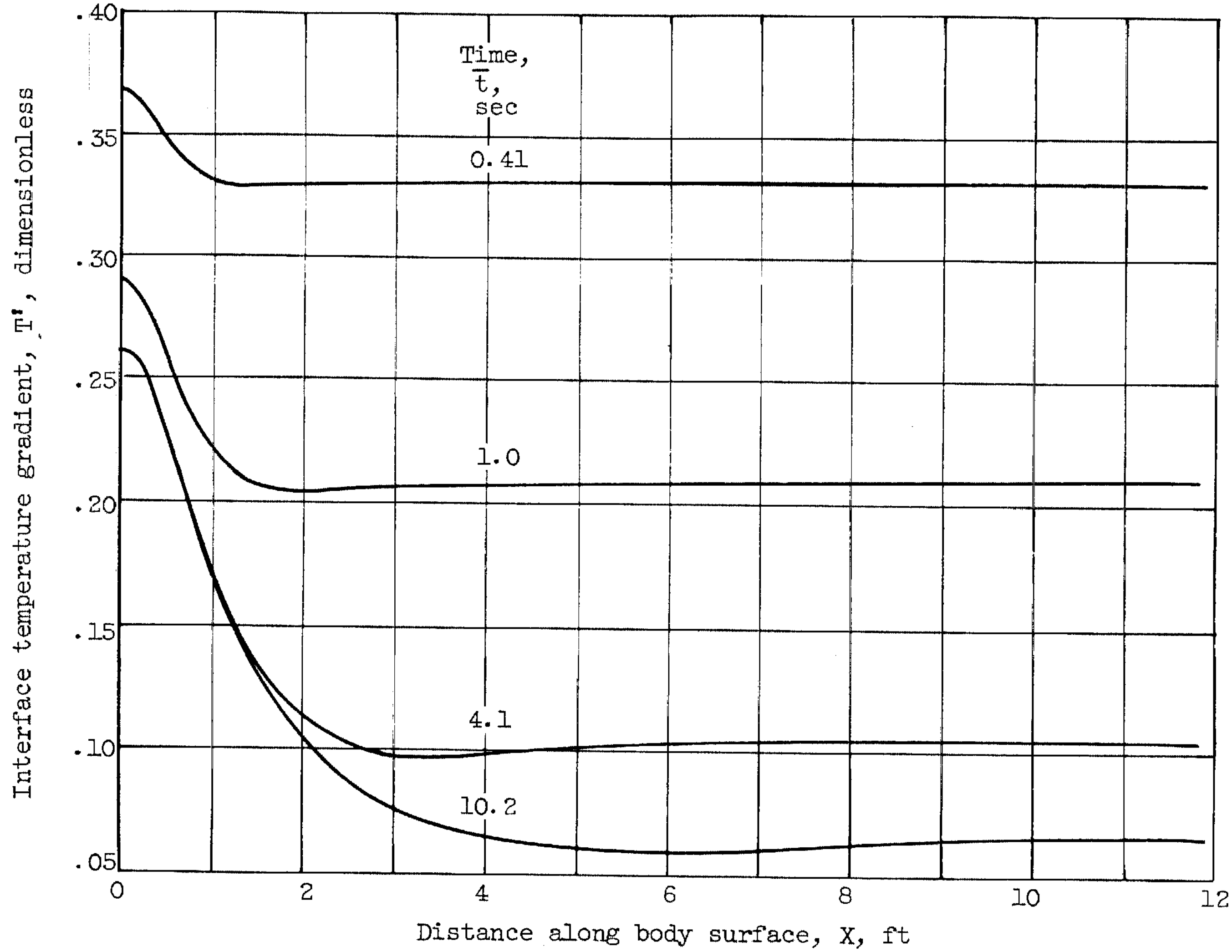


Figure 2. - Shape of ablating body.



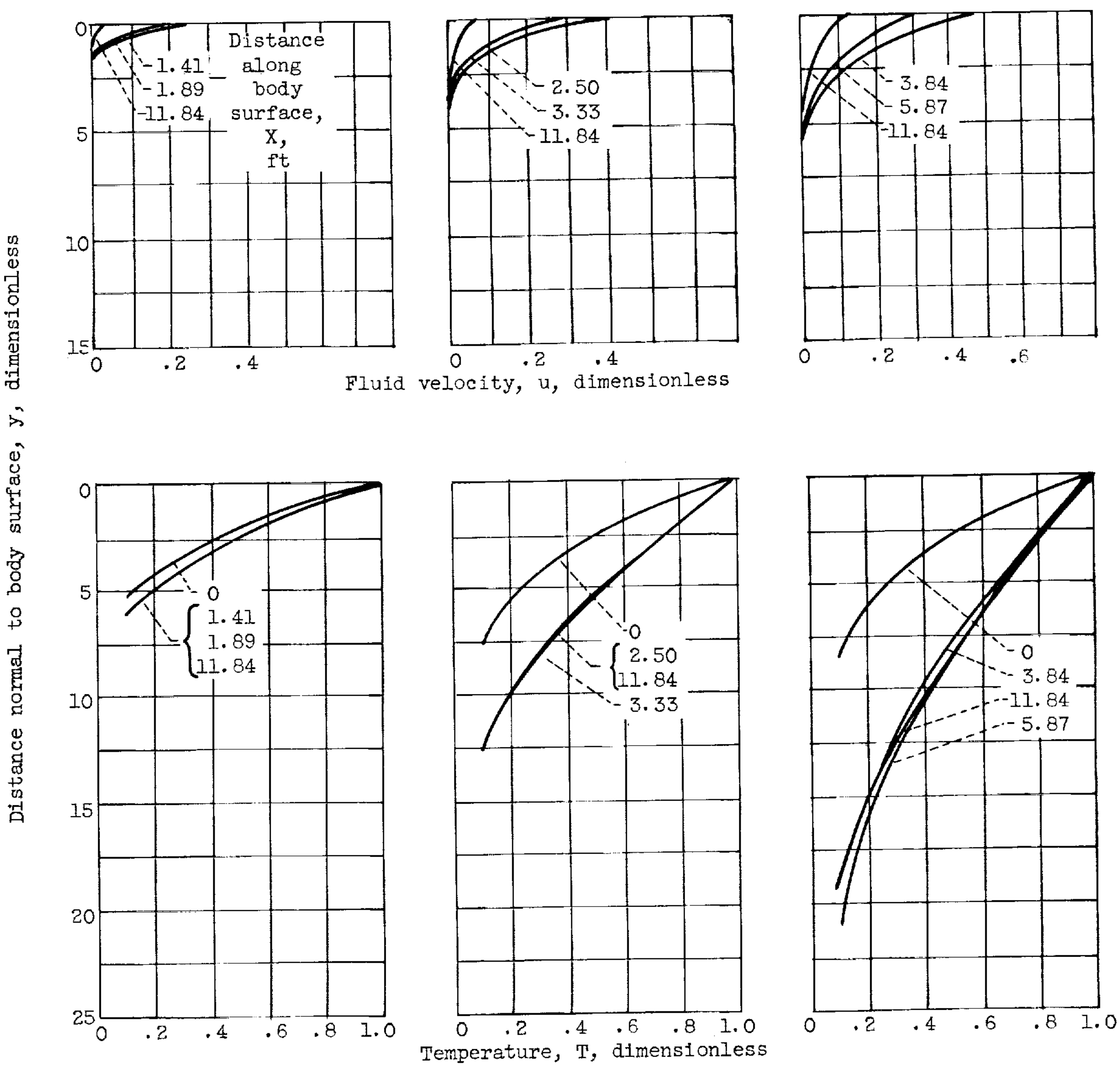
(a) Normal velocity distribution.

Figure 3. - Interface conditions for no deceleration.



(b) Temperature gradient distribution.

Figure 3. - Concluded. Interface conditions for no deceleration.

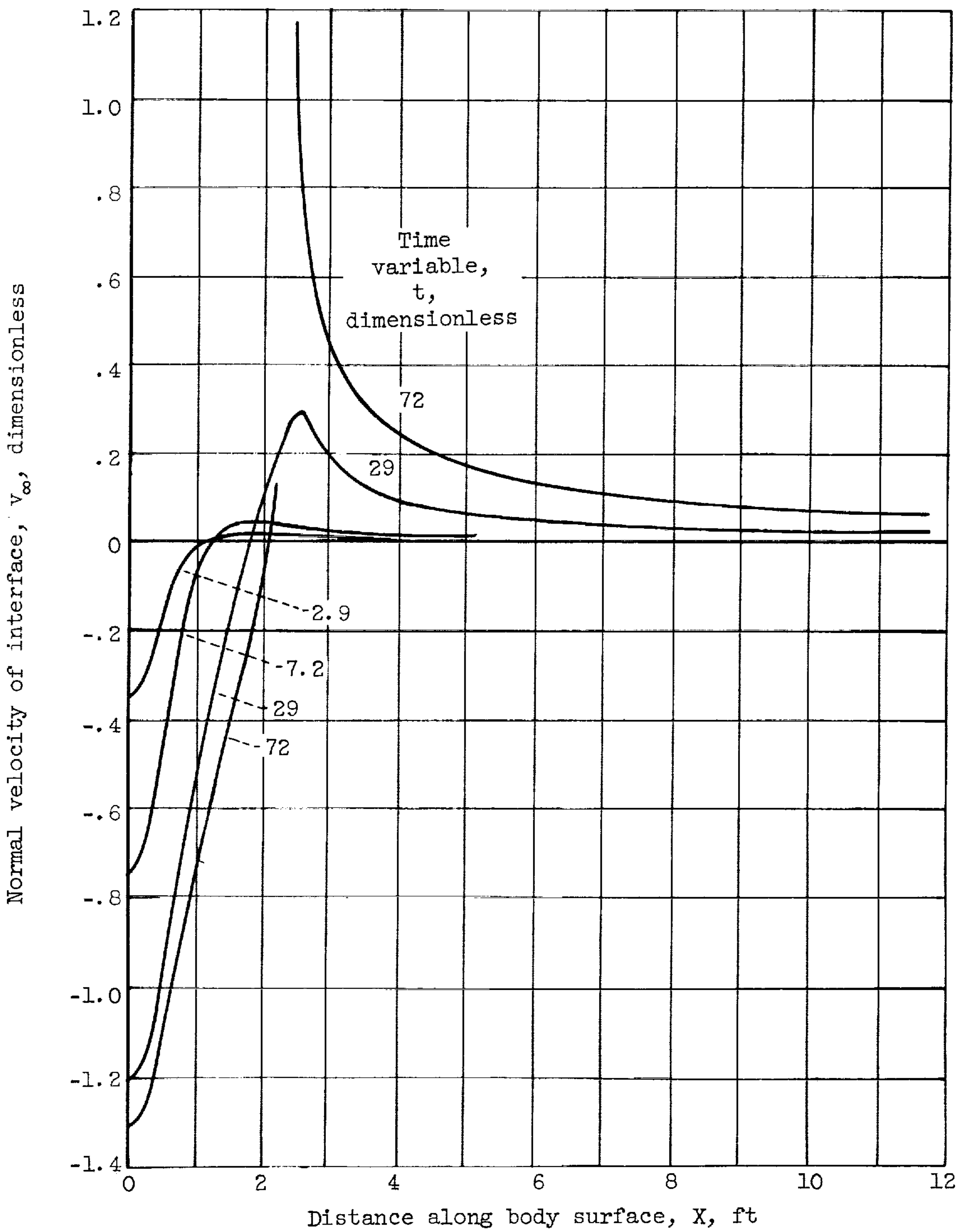


(a) Dimensionless time variable, 7.2.

(b) Dimensionless time variable, 29.

(c) Dimensionless time variable, 72.

Figure 4. - Flow-velocity and temperature profiles for no deceleration.



(a) Normal velocity distribution.

Figure 5. - Interface conditions for moderate deceleration ($g = -0.2$).

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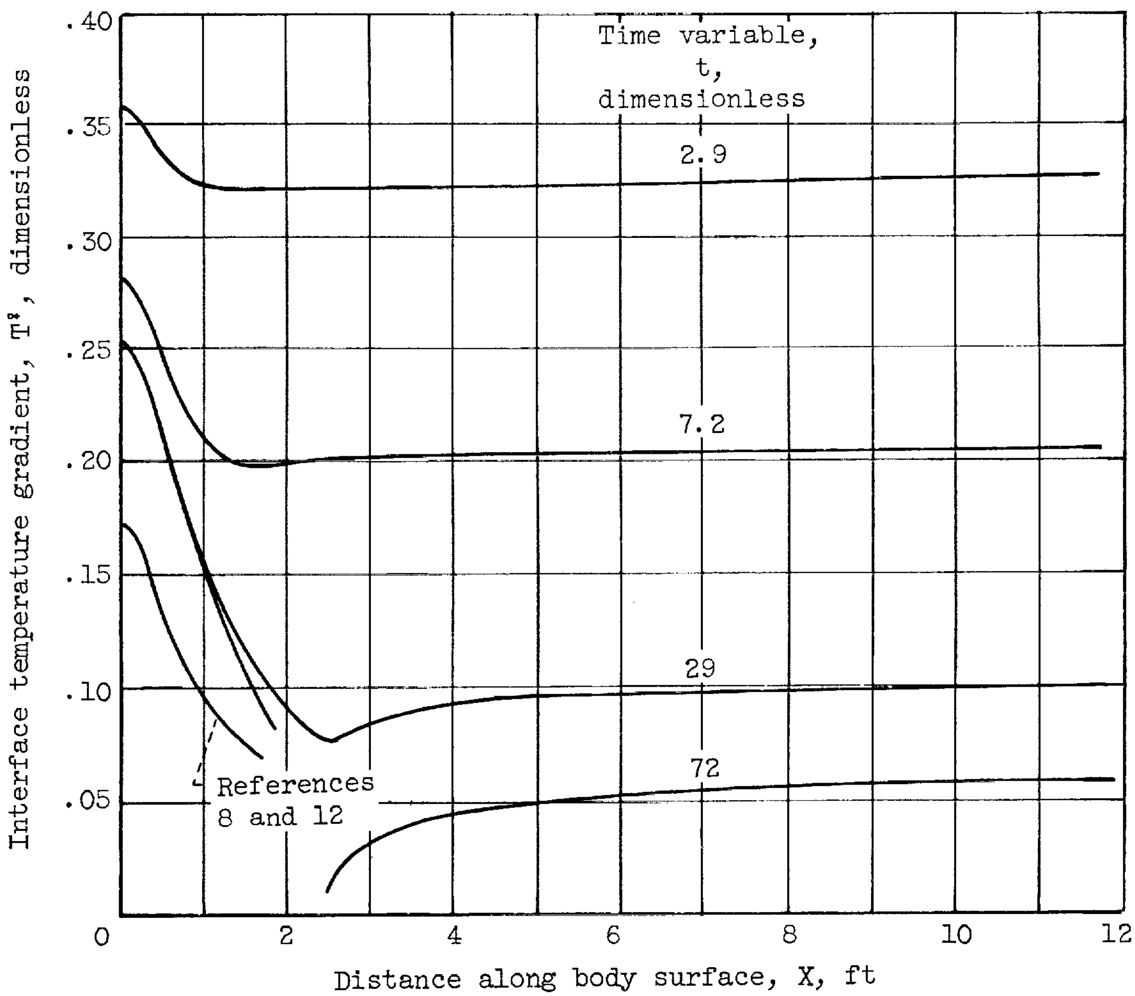
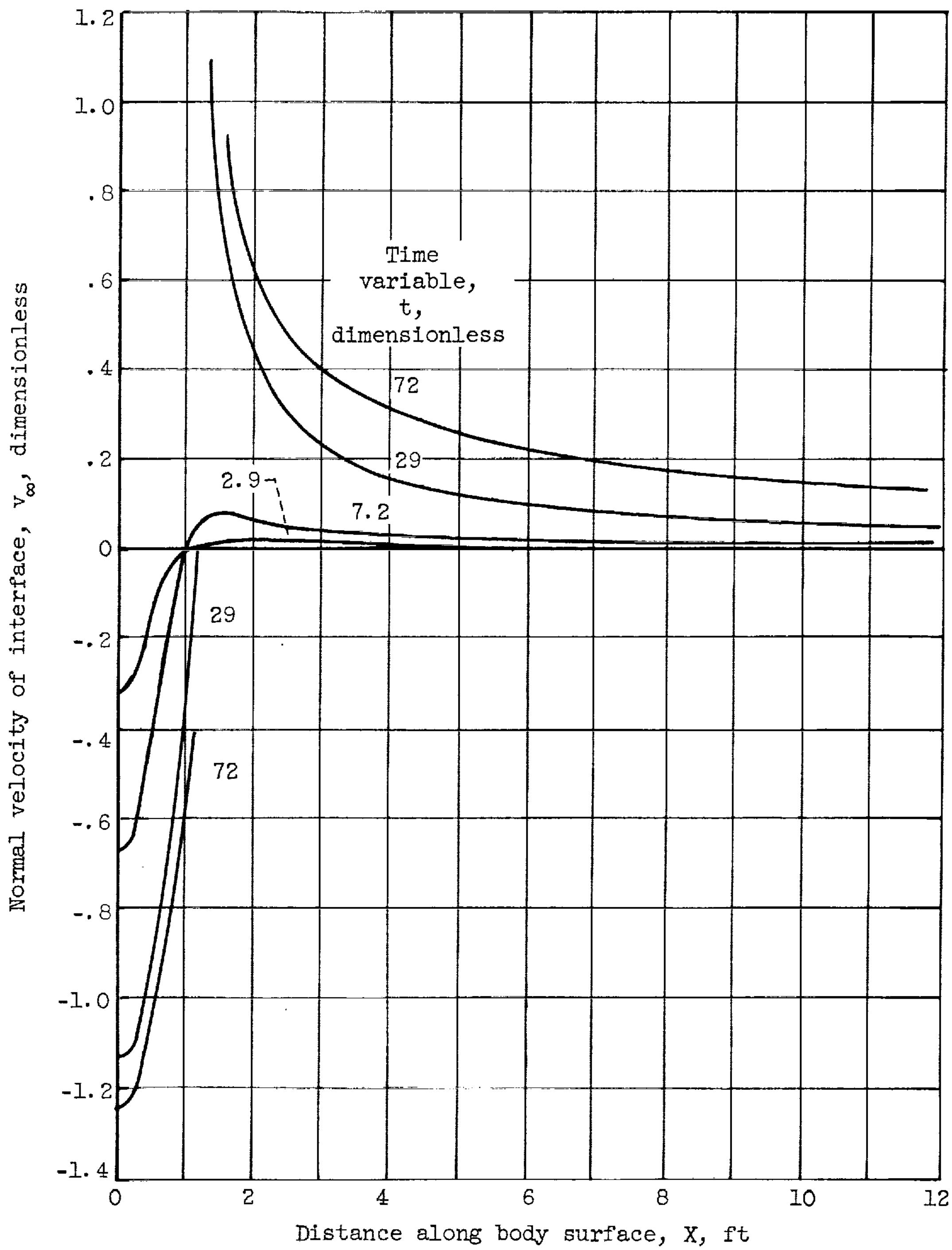


(b) Temperature gradient distribution.

Figure 5. - Concluded. Interface conditions for moderate deceleration (g = -0.2).



(a) Normal velocity distribution.

Figure 6. - Interface conditions for strong deceleration ($g = -0.6$).

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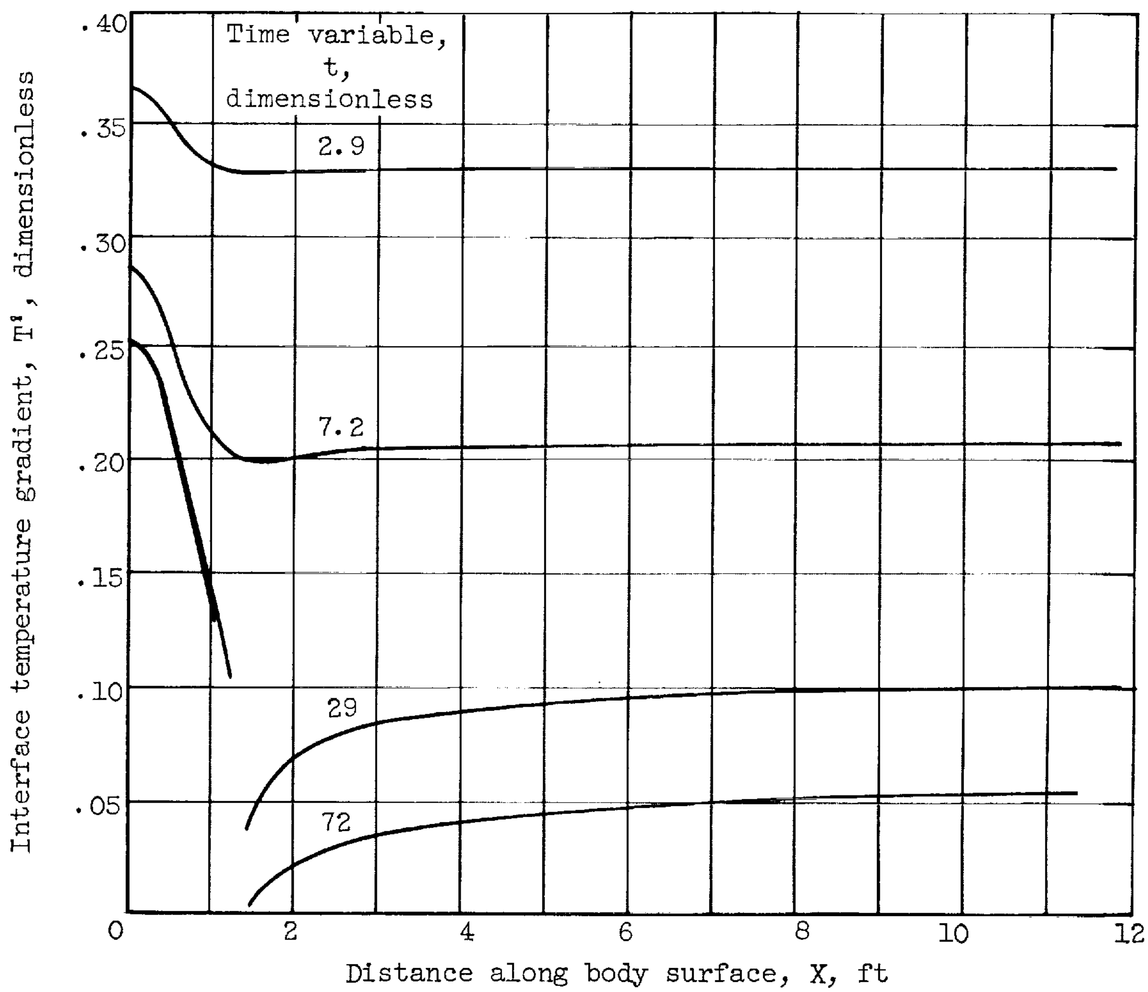
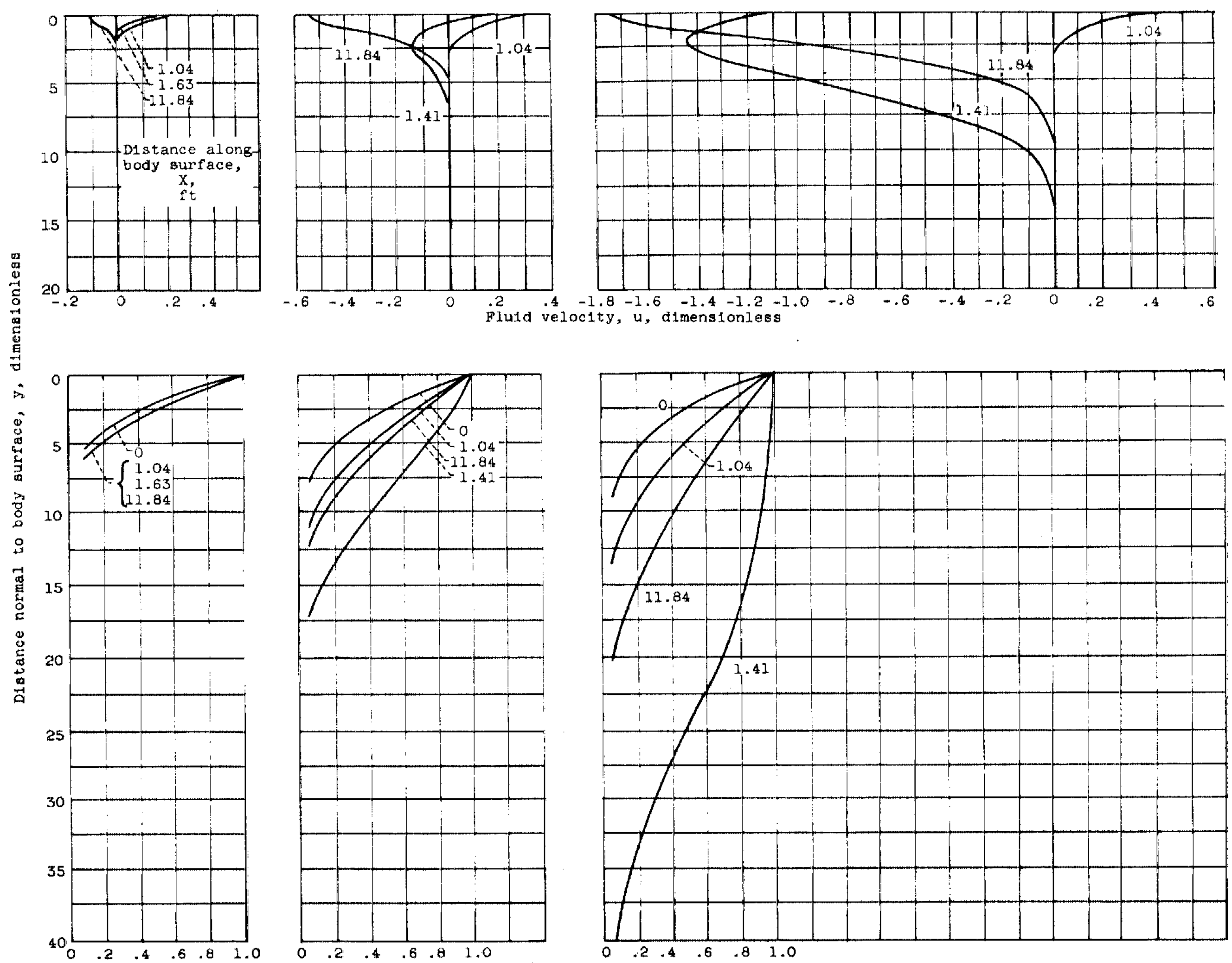


(b) Temperature gradient distribution.

Figure 6. - Concluded. Interface conditions for strong deceleration (g = -0.6).



(a) Dimensionless time variable, 7.2

(b) Dimensionless time variable, 29.

(c) Dimensionless time variable, 72.

Figure 7. - Flow-velocity and temperature profiles for strong deceleration (g = -0.6).

NASA TN D-1312
National Aeronautics and Space Administration.
ANALYSIS OF MELTING BOUNDARY LAYERS ON DECELERATING BODIES. Simon Ostrach, Arthur W. Goldstein, and Jesse Hamman. July 1962. 36p. OTS price, $1.00.
(NASA TECHNICAL NOTE D-1312)

The flow of the viscous layer of ablating material on the surface of a body of revolution entering the atmosphere was investigated primarily in regard to phenomena over the entire body in order to find the primary effect of the decelerating force on the flow and heat transfer. The phenomena were essentially unsteady exclusive of the region near the forward stagnation point. The significant dynamical parameters were determined and some solutions were obtained for various deceleration rates and times. These solutions show that deceleration causes an accumulation of ablating material in a region downstream of the stagnation point, and for this reason the thin-boundary-layer approximation will eventually fail to be appropriate.

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II. Goldstein, Arthur W.
III. Hamman, Jesse
IV. NASA TN D-1312

(Initial NASA distribution: 2, Aerodynamics, missiles and space vehicles; 5, Atmospheric entry; 20, Fluid mechanics.)

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ANALYSIS OF MELTING BOUNDARY LAYERS ON DECELERATING BODIES. Simon Ostrach, Arthur W. Goldstein, and Jesse Hamman. July 1962. 36p. OTS price, $1.00.
(NASA TECHNICAL NOTE D-1312)

The flow of the viscous layer of ablating material on the surface of a body of revolution entering the atmosphere was investigated primarily in regard to phenomena over the entire body in order to find the primary effect of the decelerating force on the flow and heat transfer. The phenomena were essentially unsteady exclusive of the region near the forward stagnation point. The significant dynamical parameters were determined and some solutions were obtained for various deceleration rates and times. These solutions show that deceleration causes an accumulation of ablating material in a region downstream of the stagnation point, and for this reason the thin-boundary-layer approximation will eventually fail to be appropriate.

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